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Socratic Seminars for Mathematics

Stephanie. I think it is graph *a* because a Ferris wheel goes round.

Tara. Graph *a* goes backward in time.

Laura. Time goes forward this way, along the *x*-axis, so it cannot be *a*.

Quavis. It goes down like this. That would make it go back in time.

Stephanie. Yes, but that is how a Ferris wheel goes.

Stephanie and Quavis gestured animatedly as they stood in front of the graphs, shown in **figure 1,** that had been drawn on the board. Tara and Laura were speaking from their desks, and the rest of their second-year algebra class watched intently. We will subsequently share more of their discussion as we discuss an example of a Socratic seminar, but we first notice how these students struggled to "orga-nize and consolidate their mathematical thinking through communication" and to "communicate their mathematical thinking coherently and clearly to peers, teachers, and others" (NCTM 2000, p. 348). The students were arguing vigorously and seemed disappointed when the bell rang to end the seminar.
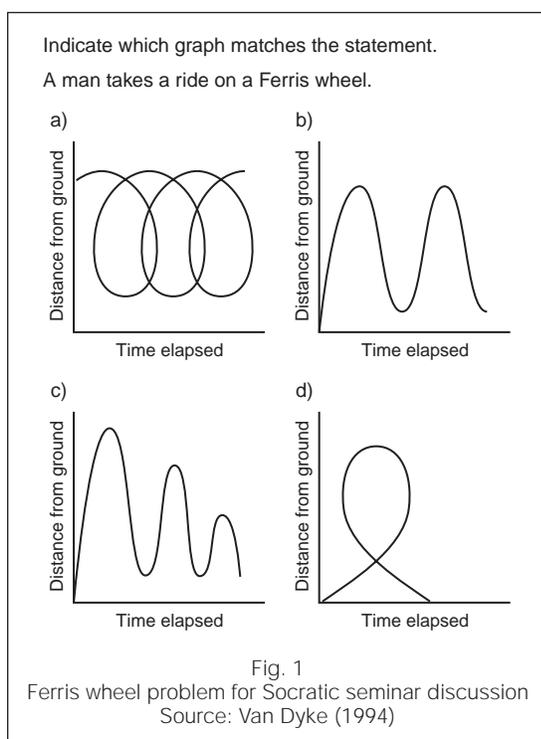
Mathematical discussions with this high level of interest and involvement are a goal of the *Standards* and are stimulating for both students and teachers. At Forest Park High School in Forest Park, Georgia, the entire mathematics department uses Socratic seminars to create classroom settings that are con-ducive to this type of discussion. Each mathematics teacher conducts several Socratic seminars a year in each class—with the whole class. For the sake of this action research, each teacher who taught second-year algebra did the seminar described in this arti-cle with at least one class, but each teacher had a control class, as well. As teachers compared stu-dents' achievement on tests that focused on the con-cept of function, they found that students who had participated in a seminar did better on the chapter test and on a posttest instrument used to measure students' understanding of the concept of function.

In this article, we share the basics of using Socratic seminars in a mathematics classroom.

BACKGROUND

For an example that indicates how Socrates taught mathematics, we turn to the dialogue between Socrates and Meno (Rouse 1956). Socrates used what we have come to call *Socratic questioning* to

Mathematics discussions with this high level of interest and involvement are a goal of the **Standards**

Indicate which graph matches the statement.

A man takes a ride on a Ferris wheel.

Fig. 1
Ferris wheel problem for Socratic seminar discussion
Source: Van Dyke (1994)

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teach the Pythagorean theorem to Meno's Greek servant boy. Socrates later said that "no one taught [the boy], only asked questions, yet he will know, having got the knowledge out of himself " (Rouse 1956, p. 50). Plato, a student of Socrates and recorder of his dialogues, called this one-on-one questioning the "art of midwifery—the art of assisting at the birth of thoughts."

John Dewey, Leonard Nelson, and Mortimer Adler have elaborated on using Socratic methods with more than one student. Dewey believed that students who were actively involved in their education learned more than those who were passive. He called his version of the seminar the *recitation*. According to Dewey (1933, p. 262), the goals of the discussion should be to "stimulate intellectual eagerness, awaken an intensified desire for intellectual activity and knowledge and love of study."

He gave several guidelines for conducting a recitation. The first was that students should be given a new problem on which to apply previously learned concepts. Then the teacher should use questioning to guide the discussion to the subject matter, with the process as important as getting the correct answer. Throughout the seminar, the line of questioning should be used to clarify and extend students' thinking. Finally, the discussion should end with a sense of closure and anticipation of what is to come next, in the form of a new problem or topic to tackle at the next seminar.

In the twentieth century, a German professor of philosophy, Leonard Nelson, examined the Socratic method and elaborated on the role of the teacher. Nelson, a contemporary of the mathematician David Hilbert, believed that the "teacher was forbidden to utter judgment in the subject matter, including the right-wrong evaluation of the students' statements" (Loska 1998, p. 238). The main role of the teacher was to ensure "a genuine mutual understanding among the students, the concentration on the respective question to prevent digression, and the preservation of the good ideas that had come up in the course of the discussion" (p. 238). Nelson believed that such a discussion, gently guided by teacher questioning, would converge on the main ideas and that any individual's errors would be challenged by other discussion participants.

Mortimer Adler's notion of seminars was firmly rooted in the progressivist tenets of Dewey. Adler (1982, p. 22) believed that the three goals in the teaching and learning process were as follows: the "acquisition of knowledge," the "development of intellectual skills" (the skills of learning), and the "enlarged understanding of ideas and knowledge." In Adler's view, the first goal was delivered through didactic teaching; the second through "coaching, exercises, and supervised practice;" and the third through the Socratic seminar.

For teaching older students by Socratic seminar, Adler proposed that classes be longer than fifty minutes. Ideally, participants in the discussion sit in a circle instead of in rows. In *The Paidea Proposal,* he explains further:

The teacher's role in discussion is to keep it going along fruitful lines—by moderating, guiding, correcting, leading, and arguing like one more student! The teacher is first among equals. All must have the sense that they are participating as equals, as is the case in a genuine conversation. (Adler 1982, p. 54)

GUIDELINES FOR USING A SOCRATIC SEMINAR

A few guidelines can help establish a respectful atmosphere for the Socratic seminar and should be discussed before the seminar begins. The guidelines that mathematics teachers at Forest Park High School found effective are as follows:

- Participants must respect one another's opinions.
- Participants do not have to raise their hands to speak, but they must not interrupt.
- Participants address their fellow classmates by name (name cards can be placed on the desks, if necessary) and should take notes.
- Participants' comments address the topic and do not digress.
- Participants settle points of disagreement among themselves. The teacher is not used as a resource.

The desks are rearranged so that students and the teacher are all seated in a circle and they can easily see one another.

The role of the teacher is to choose the topic and guide the discussion by using skillful questioning, including questions that focus the discussion on the topic, prevent any improper comments or behavior, and clarify important concepts brought out in the discussion. The teacher does not state his or her position on the correctness of the viewpoint of the students during the seminar.
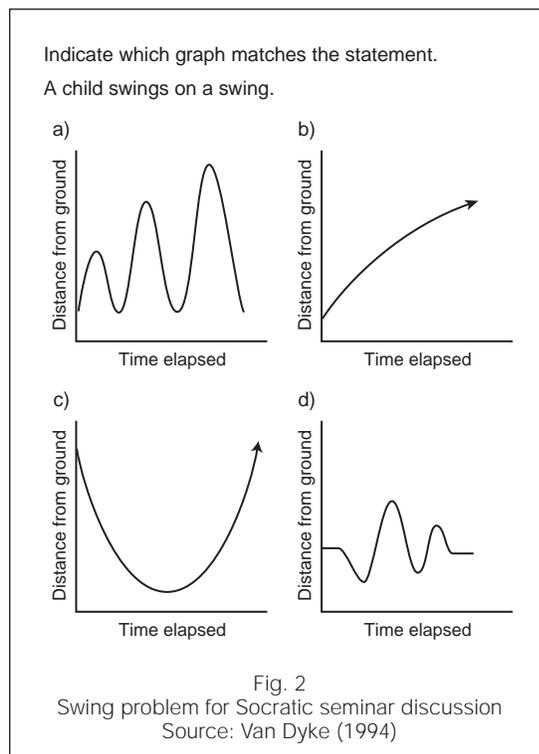
QUESTIONS AND PROMPTS

Thought-provoking mathematical problems or questions are critical to the success of the seminar. We found some problems from a *Mathematics Teacher* article (Van Dyke 1994) to be very effective in provoking a rich and lively discussion about important mathematics. NCTM Addenda documents and the Illuminations Web site are also useful resources for good problems. The problems cannot be too easy; they need to involve interesting mathematics ideas that students struggle to understand. Each seminar used three of Van Dyke's problems: the swing problem first, next the train problem, and then the Ferris wheel problem.

Thought-provoking mathematical problems are critical to the success of the seminar

AN EXAMPLE SEMINAR

The goal of this seminar was to clarify students' understanding of the definition of *function.* Eight second-year algebra classes at Forest Park High School used a Socratic seminar to address this objective. The teacher initially asked students to define *function* on notebook paper, and they then discussed their definitions as a group. The teacher asked students to share their definitions, when needed, during the seminar so that any misconceptions that may have been discovered through the discussions could be clarified. Then the teacher distributed several choices of graphs to represent each scenario. Each student decided on his or her own answer before the discussion began. For example, the first question asked the students to decide which graph in the group shown in **figure 2** best illustrated the scenario of a child swinging on a swing.

Some students literally interpreted the graph as a visual representation of the motion of the swing

Indicate which graph matches the statement.

A child swings on a swing.

Fig. 2
Swing problem for Socratic seminar discussion
Source: Van Dyke (1994)

Deidre. I chose *a* because when you swing back and forth, the longer you swing, the higher you go.

John. You do not start in the air though. You have to get on the swing.

Quavis. It says the child swings—the kid is already on the swing. Anyhow, you know that you start from a higher point because someone holds you up. That's where you start. Graph *a* doesn't show you starting high enough above the ground.

Stephanie. You don't always start with a push.

Tara. Graph *b* could represent one push starting lower from the ground.

Deidre. Yeah, but it says *swings,* meaning you go back and forth.

Sam. Yes, I agree.

This type of conversation was typical of the ones in each of the second-year algebra classes. Deidre confidently explained her position that graph *a,* which was the correct answer, fit the given context. The discussion that followed focused on the starting time and the distance from the ground. Some students initially seemed to attend only to particular points on the graph, such as the starting time $x = 0$ and the initial distance from the ground. However, Deidre continued to justify her thinking by using the whole graph and the functional relationship $f(\text{time}) = \text{distance}$.

Quavis. The arrow represents that it goes on and on, that the graph continues.

Deidre. Graph *a* shows that it goes up and down like the back and forth.

Tara. Look at *b.* Doesn't it look like the first hump in *a* is a ray?

Quavis. We need to look at elapsed time. It looks like at that time, it stops at the humps so that cannot be right. I think graph *c* shows it best because someone pulls the kid back and lets him go, and the arrow says that "it keeps going."

Laura. But the swing always starts straight down; how does the swing get up like in *c?*

Quavis. Somebody had to pull you back, and that is where *c* starts.

This excerpt was typical of the seminars in that some students, for example, Quavis, interpreted the graph literally, as a visual representation of the motion of the swing. When he explained that the arrow indicated that it went on and on, he illustrated the movement with his hand, repeating the line of the graph in *c.* Deidre continued to argue for her initial interpretation, but she was not able to convince the other students. Tara suggested another approach but was ignored in the passion of the discussion, and the discussion returned to graphs *a* and *c.*

Jarell. I don't like *b* and *c.* I like either *a* or *d* because *b* and *c* are rays and *a* and *d* are functions.

Quavis. You are missing my point. I said if you think of it as a ray, then the arrow signifies that it doesn't stop—it keeps going.

Tony. A ray goes indefinitely in one direction. It doesn't mean that it comes back down.
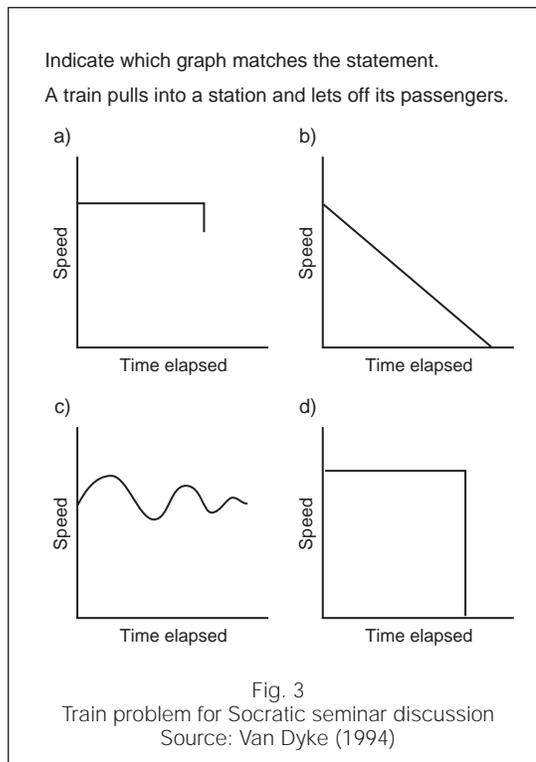
Jarell. Yeah, so it means that it can keep on swinging.

This dialogue illustrates another typical scenario, in that several people argued for their own misconceptions in their mathematical understanding. In retrospect, it might have been an opportunity for someone to ask Jarrell to clarify what he meant by *ray* and why it mattered whether or not the graph was a *function.* In this excerpt, Quavis continued to try to persuade others that graph *c* represents a child swinging on a swing and that the arrow on the graph indicates that the child will keep swinging. Students who understood which graph represented the situation were sometimes frustrated with their peers who did not understand. These students often glanced to the teacher for help but then realized that convincing their peers was their own responsibility. They reentered the discussion with renewed determination to explain their correct mathematical reasoning more precisely and persuasively. Their explanations helped other students revise and refine their mathematical thinking.

In this seminar, the teacher redirected the discussion on two occasions. In her first redirection, she encouraged the students to discuss the quantities associated with the *x*- and *y*- axes. Student discussion then brought out that the distance was a function of time and suggested that function notation be used. In her second redirection, the teacher asked which graphs represented functions. These probes aided in the teacher's assessments of the students' functional representations and extended their discussion to a higher level of thinking. On reflection, the teacher wondered whether students truly understood the connections between their mathematical knowledge of functions and their application to phenomena in the physical world.

After a consensus was reached, the teacher posed the second problem, which asks students to determine the graph that represents a train pulling into a station and letting off its passengers. Students were to choose the graphical representation in **figure 3** that best matched the scenario. (The correct answer is *b.*) In the eight seminars that used this train problem, students focused on the end behavior of the graph and laughingly reenacted how it would feel to be on train *d,* which stopped suddenly; on trains *a* or *c,* which did not stop for passengers to disembark; or on train *c,* which alternately accelerated and decelerated. This problem and students' reenactments helped them grapple with the dependence of speed on time. Typically, the discussion of the train graph did not take as long as either of the other two discussed here.

The last problem, shown in **figure 1,** represented a man taking a ride on a Ferris wheel. (The correct answer is *b.*) This problem is designed to help students deal with a common conceptual error, that is, mistaking the motion of the event for a graph of its distance and time (Dugdale 1993). When the stu-

Indicate which graph matches the statement.

A train pulls into a station and lets off its passengers.

Fig. 3
Train problem for Socratic seminar discussion
Source: Van Dyke (1994)

dents initially shared their ideas regarding the graph that matched the Ferris-wheel problem, their answers were split evenly between graph *a* and graph *d,* with a very few *b* or *c* answers.

Later in the discussion, the students tried to convince Stephanie and eight or nine other students that graph *a* is not a function. They tried to explain their reasoning using the line of the graph and what it represents. However, Stephanie and other students argued adamantly that graph *a* represented a ride on a Ferris wheel for that very reason, that the graph represents how a Ferris wheel goes around. They did not seem aware of the functional relationship between distance and time.

Tony. I think it's *d.*
Donald. I am with Tony. A Ferris wheel doesn't go up and down. It goes around.
Laura. It's a circle. . . . The height of the Ferris wheel stays the same. It never changes, and *b* shows that it goes the same height.
Tara. That is not what the graph is saying. Graph *b* is saying that you go up. Then it is going forward. It has a definite shape—it peaks and represents the highest point. A Ferris wheel goes around, and graph *a* shows the movement you feel.
Patrick. There is no answer to this one.
Tara. You start at ground level and go up. I'm talking from firsthand experience. You first get up on a pedestal—that's ground level. See how that happens in *a?*

Students focused on the end behavior of the graph

Kia. I agree with Tara. You are not touching the ground.

Quavis. No, I agree with how you start, but I look at *d* and it shows the whole story. You walk up, get on the pedestal, get in a seat and go up, and then it goes around.

Stephanie. So it is *d.* I can see that. You still go around, not up and down.

At this point (late in the seminar), a lot of the students were still trying to make sense of the graph and the Ferris-wheel context. They wanted the graph to illustrate literally the movement they felt on a Ferris wheel. Although some students were confident and vocal in the beginning about *a* and *d* not being functions because "they are going back in time," this discussion briefly fell by the wayside while students struggled with their misconceptions.

Quavis. No! That would be backward in time again.

Stephanie. See; this is how Ferris wheels go. [*Traces the path of d in the air.*]

Laura: You are trying to make this the track—it is not the track. Okay, look at this. [*She goes up to the board and draws a copy of d.*] Look at this point where the lines cross. See, you cannot be in two places at once! [*She draws a straight line from the cross to the loop.*] So it cannot be *d* or *a.*

Quavis. Yeah, and *d* and *a* are not functions because they do not pass the vertical-line test.

In this last excerpt, Laura uses the intersection of two parts of the graph in *d* and the fact that "you cannot be in two places at once" to show that the line of the graph is not the track. In doing so, she reminds Quavis and others of the vertical-line test for showing whether a graph is a function. Most of the class verbally agreed that graph *b* fit the Ferris wheel context. Further, they seemed to be able to make sense of the functional relationship that the graph illustrates. However, Stephanie and one other student still were not convinced and left that day's seminar without resolution. The teacher took notes and planned to follow up with them. She also decided that the next day's lesson would include a discussion of why it was important that these graphic interpretations of the relationship between time and distance involved in different events (swing, train, and Ferris wheel) should be functions. In the discussion at the beginning of this article, Tara and Quavis both understood that graphs *a* and *b* show "going back in time," and Laura suggested using the vertical lines to test whether the graphs of the Ferris wheel were functions, but neither the teacher nor the teacher-educator observers could determine whether students understood that graphs in which time is the independent variable must be functions.

REFLECTION

We found that the method of Socratic seminars was very effective in encouraging students to assume responsibility for reasoning and communicating convincingly about mathematics. Student and teacher feedback confirmed that all participants, especially those who appeared to be more verbal learners, found that the seminars were fun. Those students showed understanding that they had not shown on pencil-and-paper assessments. The seminar helped the teacher assess students' conceptual understanding of functions and their graphical representations, in addition to providing a forum for rich discussion of an important mathematical topic. The teachers and two teacher educators who observed the eight seminars were surprised at how literally the tenth- and eleventh-grade students tried to make sense of the graphical situations presented, because most of these students were able to solve typical textbook problems relating to functions with relative ease. The Socratic method provided rare opportunities for the participating students and teachers. Students had a forum for articulating and organizing their mathematical understanding; meanwhile, their teachers could focus on listening to and reflecting on students' understanding.

The six Forest Park High School teachers and the two teacher-educator observers were all impressed with the results of the Socratic seminar in the eight classes that tried them. The assessments used after the seminars showed that students who participated understood the concept of function better than students who had not participated in a seminar. Our observations of the seminars indicated that the students were actively involved in reasoning and communicating about mathematics and explaining functions to their classmates who had misconceptions. We saw repeatedly that when students discussed their ideas with others, they continued to revise, refine, and improve them (Borasi 1992; Moschkovich 1998). Consider adding the Socratic seminar to your teaching repertoire as a productive and entertaining way to promote your students' mathematical reasoning and communication skills.

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